San Diego Math Circle - Fermat
September 29, 2012

Fuggedaboutit! An Introduction to Taxicab Geometry

Answers

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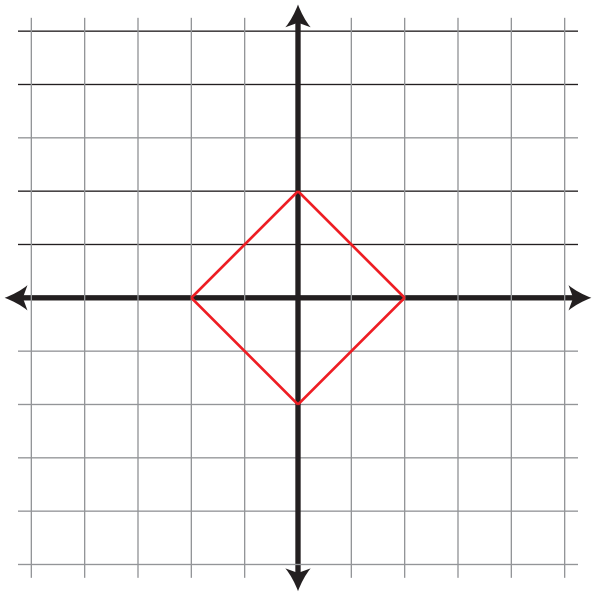
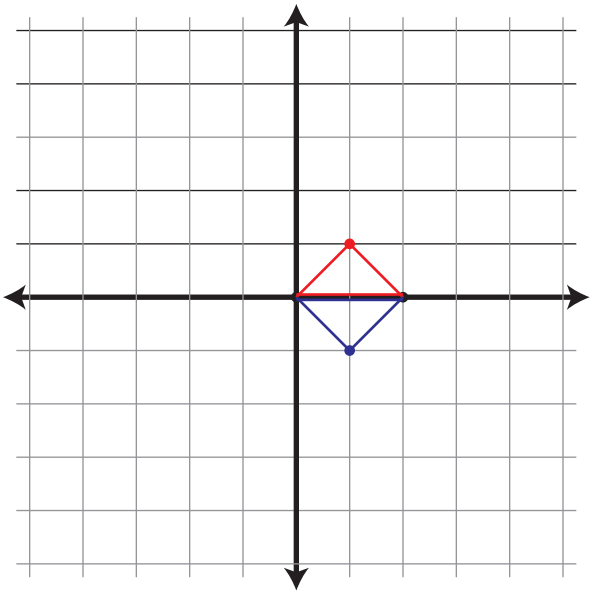
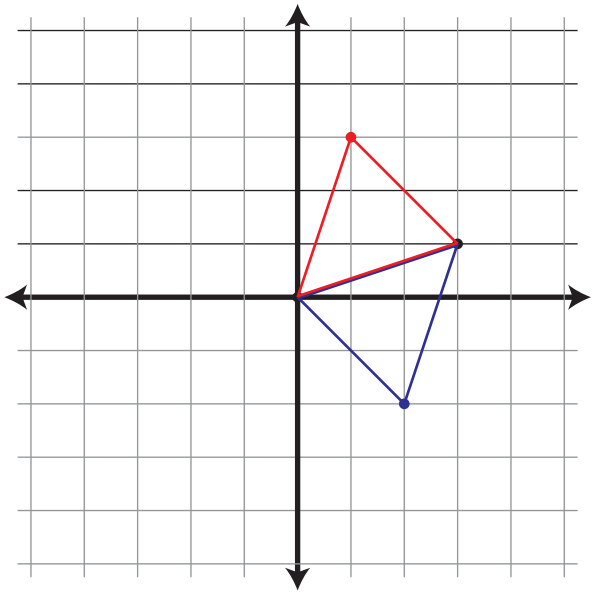
What does a **taxicab circle**, centered at the origin $(0, 0)$, with radius 2 look like?



Find the third point of the **taxicab equilateral triangle** whose other two points are $(0, 0)$ and $(2, 0)$.



Find the third point of the **taxicab equilateral triangle** whose other two points are $(0, 0)$ and $(3, 1)$.



Triangle Inequality

The **triangle inequality** states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side (for all triangles). Is this still true if **taxicab length** is used instead? Under what situations is it true or untrue?

Let the three vertices of the triangle by $A : (x_1, y_1)$, $B : (x_2, y_2)$, and $C : (x_3, y_3)$. If

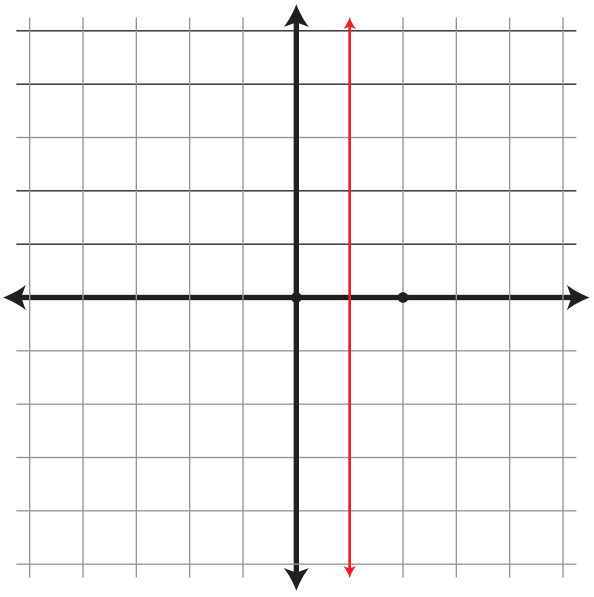
$$x_1 \leq x_2 \leq x_3$$
$$y_1 \leq y_2 \leq y_3,$$

then the **taxicab distance** from A to B and the taxicab distance from B to C add up to the taxicab distance from A to C. Additionally, the angle at B is right or obtuse.

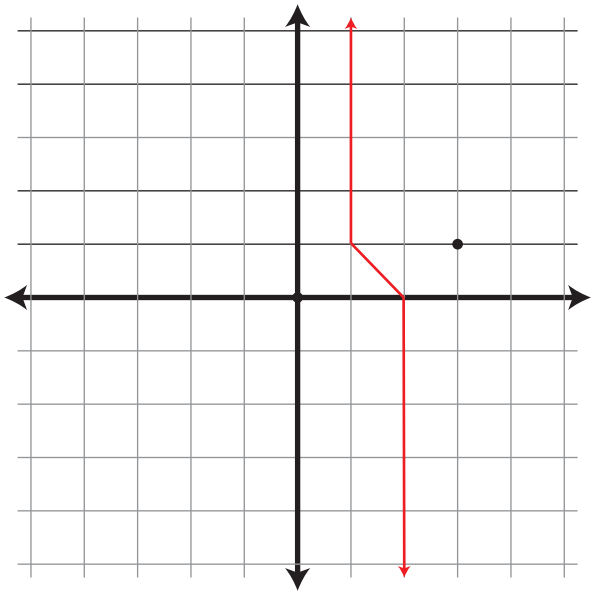
Equidistant Lines

Suppose we wanted to find the set of points that are always the same distance to two points A and B. For example, let $A = (0, 0)$ and $B = (2, 0)$. Then the set of points that are **equidistant** to both A and B is the line $x = 1$. It turns out that this line will always be the perpendicular bisector of the line segment $\overline{AB}$. What happens if we use **taxicab distance**?

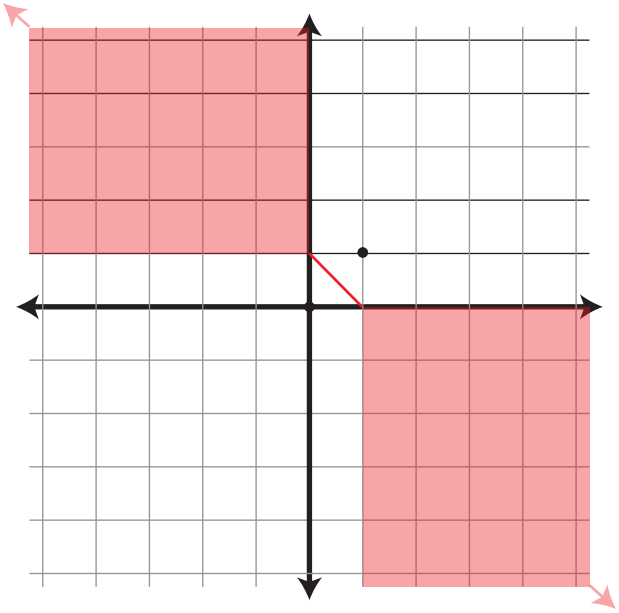
Find the set of points that are the same **taxicab distance** to $(0, 0)$ and $(2, 0)$.



Find the set of points that are the same **taxicab distance** to $(0, 0)$ and $(3, 1)$.



Find the set of points that are the same **taxicab distance** to $(0, 0)$ and $(1, 1)$.

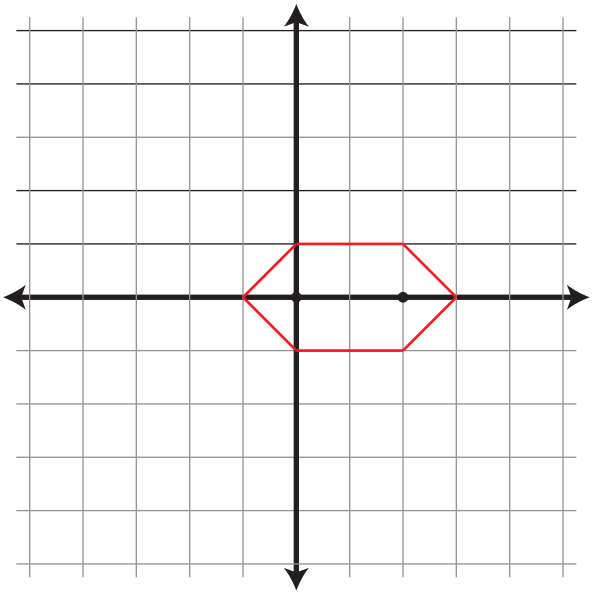


Ellipses

An ellipse is the set of points for which the sum of the distances to two points is the same. The two points are known as the **foci** (plural of focus). A circle is a special form of ellipse where the two foci are the same point, the center of the circle.

In a **taxicab ellipse**, the sum of the **taxicab distances** to the foci is the same.

Find the set of points for which the sum of the **taxicab distances** to $(0,0)$ and $(2,0)$ is equal to 4.



Find the set of points for which the sum of the **taxicab distances** to $(0,0)$ and $(3,1)$ is equal to 6.